Two Become One: Theory and Experiment

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Conclusions

- Experiments have revitalised theory of SAT
  many open problems remain
- 'Empirical science of algorithms' developing
  theory helps refine models
- Theory & Experiment can form a virtuous cycle

Map of talk

- Grovelling apologies for Spicey title
- The SAT Phase Transition
- Empirical Science of Algorithms
- The Virtuous Cycle: Case Studies

Can't Get No Satisfaction

- Brian Hayes, *American Scientist*, March-April 1997
- An example 2-SAT problem

invite Peru or not invite Qatar	$P \vee \neg Q$
invite Qatar or invite Romania	$Q \vee R$
not invite Romania or not invite Peru	$\neg R \vee \neg P$

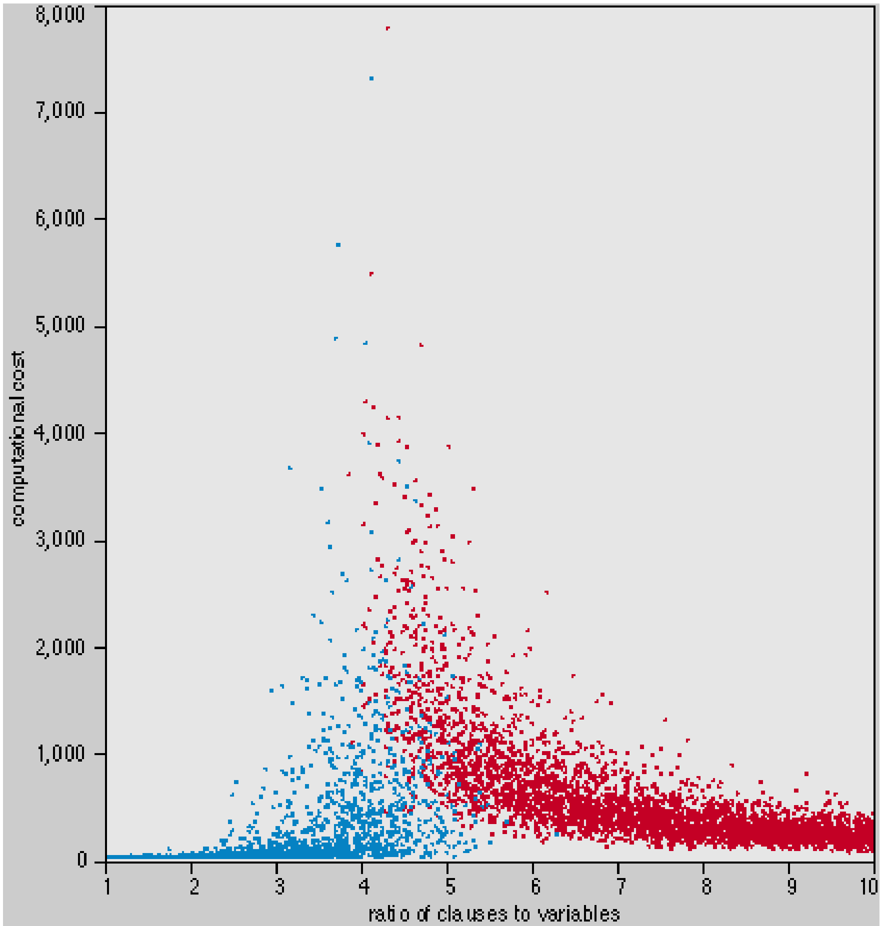
- SAT automation due to William Stanley Jevons 1869
- standard backtracking method due to Davis/Putnam/Logemann/Loveland 1962

The SAT Phase Transition

- Complexity analysis by Goldberg 1979, Purdom, Franco, ...
  - proved many average polynomial-time results for DP
  - where are the really hard NP problems?
  - occasional hints (Brown & Purdom 1981)
- Empirical studies by Cheeseman 1991, Selman, Mitchell ...
  - hard problems at 'phase transitions'
  - analogy with physics (water, magnets, ...)
  - many studies of Random 3-SAT

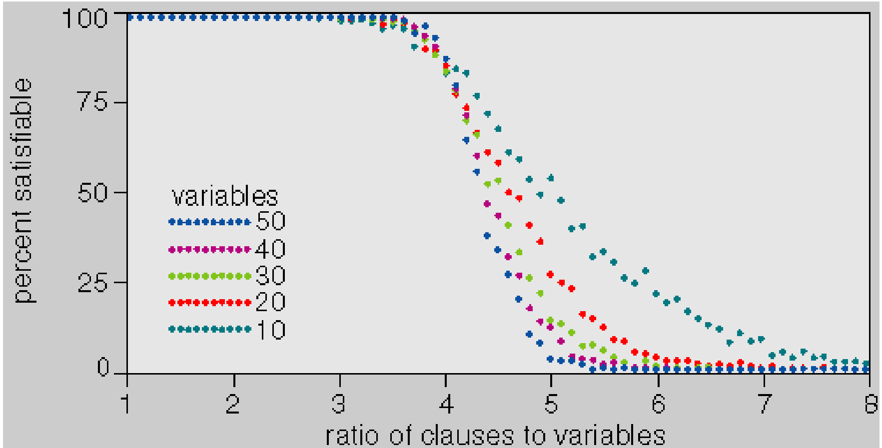
Easy-Hard-Easy

Figures from Hayes
10,000 SAT Problems
35 variables
Blue dots satisfiable
Red dots unsatisfiable



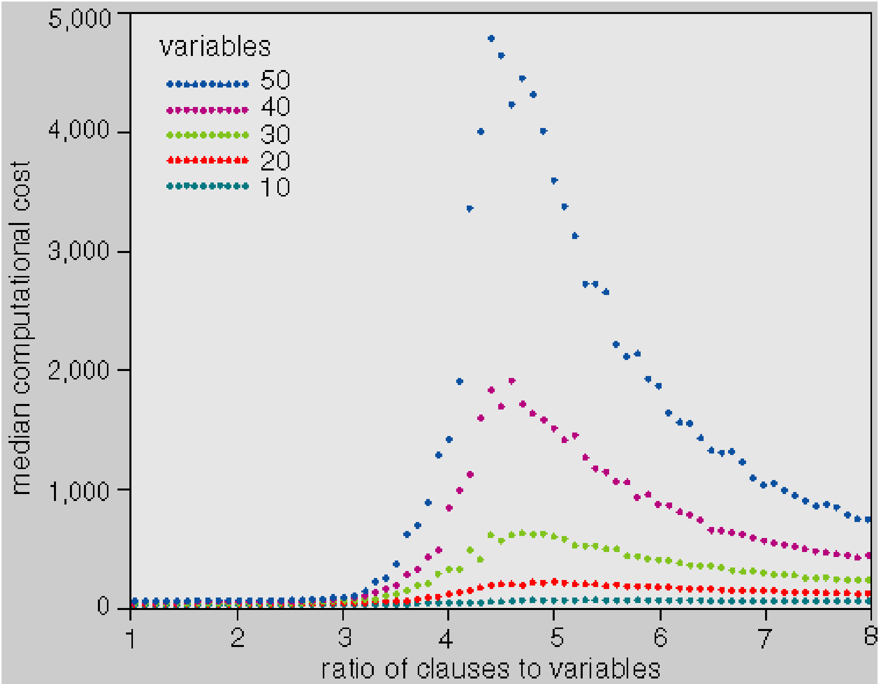
Sharpening Phase Transition

Each dot
= 300 problems



Sharpening Phase Transition (2)

Each dot
= 300 problems



Asymptotic Theory

- hard problems for resolution when c *greater than* $5.2n$

  Chvatal & Szemeredi 1988

- phase transition in 2-SAT at $c = n$

  Goerdt 1992, Chvatal & Reed 1992

- 50% sats when $3.0n < c < 4.6n$

  Frieze & Suen 1993, Kirousis, Kranakis, Krizanc 1996

- phase transition of zero width

  Friedgut 1997 (?)

Open Questions

Q Are problems at the phase transition hard?

Q Is the asymptotic transition at a fixed point?

Q If so, where is it?

Q How does transition scale at finite sizes?

Interlude

```
(325 dp (T 1 0))
(326 dp (T 1 0))
(327 dp (T 1 0))
(328 dp (T 1 0))
(329 dp (T 1 0))
(330 dp (T 1 0))
(331 dp (T 2 1))
(332 dp (T 1 0))
(333 dp (T 1 0))
(334 dp (T 3 2))
(335 dp (T 350163776 62))
```

Interlude (2)

How many digits in the number `3...6` in the raw data?

- 6?
- 7?
- 8?
- 9?
- 10?

An Empirical Science of Algorithms

Title proposed by Hooker 1994, *Operations Research*

- Algorithms are formal structures

  does not mean we *cannot* do experimental science

- Treat algorithms as natural objects

  we have wonderful experimental apparatus for their study

  undergraduate labs $\approx$ quintillion cycles/weekend

- Empirical science involves *theory*

  compare Quantum ElectroDynamics (QED)

An Empirical Theory

Theoretical Computer Science

- link of informal notion ‘computer’ with formal one
  based on Church-Turing thesis
- dependent on empirical observation
  physical computers are Turing machines
- subject to falsification
  quantum computers

An Empirical Theory (2)

Formal Definition of Constrainedness

- link of informal notion ‘constrainedness’ with formal one
  based on formal definition κ
- dependent on empirical observation
  κ generalises phase transition parameters
  κ can be used in constrainedness-heuristics
- subject to falsification
  does not match asymptotic HC phase transition
  Frank, Gent & Walsh 1998

Theory-Experiment Virtuous Cycle

- In any experimental science we expect . . .

  theory to help experiment . . .

  . . . to help theory to help experiment . . .

  . . . to help theory to help experiment . . .

- Case studies since 1996 suggest the cycle is developing.

Gent & Walsh

Paper in *Computational Intelligence*, 1998

- Ian suggested merged algorithm 'Greedy + CKK'
- Toby experimented

  never found any difference
- Toby theorised

  proved that Greedy + CKK = CKK

Theorist and Experimenter one and the same

Smith & Dyer

Paper in *Artificial Intelligence*, 1996

- Barbara experimented on random CSP's
  conjectured model of behaviour
- Martin analysed model theoretically
- Experiments showed:
  convergence to predicted values
  link of phase transition to model

Theorist and Experimenter co-authors

Achlioptas, Kirousis, Kranakis, Krizanc, Molloy, Stamatiou

Paper in Constraint-Programming 1997.

- Many experiments on random CSP's from 1994
- Dimitris et al found asymptotic flaw in model
  limiting phase transition disappeared
- Followup experimental paper from APES to CP-98 (?)

Theorists and Experimenters at same conference

Schrag & Crawford, Dunne & Bench-Capon

Papers in *Artificial Intelligence*, 1996, 1997

- Bob & Jimi experimented on prime-implicates

  conjectured result for $n = 9$

- Paul & Trevor theorised

  proved conjecture

  generalised and proved exact result for all n

Theorists and Experimenters in same journal

Caldwell, Gent & Underwood

Paper submitted to *Theoretical Computer Science*

- Judith did type-theoretic proof of algorithm
- Jim did Nuprl proof and extracted program
- Ian experimented with Scheme code

  novel algorithms not experimented on before

Tight cycle? Runs of program used to inform proof.

Loose cycle? Ian & Jim need Judith to translate.

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